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ABSTRACT

of the dissertation for the degree of Doctor of Philosophy

WAVES IN A DEFORMABLE PIPE FILLED WITH VISCOUS BUBBLY LIQUID

Specialty:	2003.01 – Fluid, gas and plasma
Field of science:	Mathematics
Applicant:	**Reyhan Sayyad kizi Akbarly**

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The work was performed at the department of "Theoretical mechanics and continuum mechanics" of the Baku State University.

Scientific supervisors: Doctor of Phys.-Math. sc., prof.
Rafael Yusif oglu Amanzade
Candidate of Phys.-Math. sc., ass. prof.
Yusif Mammadali oglu Sevdimaliyev

Official opponents: Doctor of Science in Math., sc., prof.
İlham Teymur oglu Pirmammadov
Doctor of Phys.-Math. sc., prof.
Fuad Seyfaddin oglu Latifov
Doctor of Science in Math, assoc. prof.
Azad İsa oglu Gurbanov

One time Dissertation council BFD 2.17/1 of Supreme Attestation Commission under the President of the Republic of Azerbaijan operating at the Baku State University.

Chairman of the one time Dissertation council:
Full member of ANAS, Doctor of Phys.-Math. sc., prof.
________________ **Magomed Farman oglu Mekhdiyev**

Scientific secretary of the one time Dissertation council:
Doctor of Science in Mechanics, ass. prof.
________________ **Laura Faig kizi Fatullayeva**

Chairman of the one time scientific seminar:
Corresponding Member of ANAS, doctor of phys.-math. sc.
________________ **Rauf Haji oglu Gardashov**

GENERAL CHARACTERISTICS OF THE WORK

Rationale and development degree of the topic. The effect of the multiphase of the environment makes it difficult to study the hydrodynamic processes and movements that take place in this environment. This manifests itself in the propagation of waves generated by sharper oscillation and impact effects. The propagation of excitation waves in the flow of bubbly liquids in the pipe was studied. It is of exceptional importance in the study of the regularities of these processes, modern technologies for the creation of power plants, as well as the development of new methods in military work and the creation of a scientific basis for their analysis. The study examined the wave motion in many areas of energy, petrochemical processes, oil refining, which is characteristic of cases of one-dimensional stationary gas-liquid flow. Research on the hydrodynamics of heterogeneous environments from the standpoint of whole environment mechanics usually discusses the results of many branches of physics, including shock wave physics, gas dynamics, explosion physics, hydraulics, thermal physics, and filtration theory. The topic of the represented dissertation work is related to the solution of this problem. In this sense the topic of **the dissertation work is actual.**

Object and subject of the study. Propagation of small excitation waves generated by a monochromatic source in a thin infinite circular cross-sectional cylindrical shell containing a two-phase medium consisting of a mixture of liquid and gas phases.

Goal and objectives of the study. The main purpose of the dissertation is to study the characteristics of the evolution of waves in a hydroelastic system consisting of a cylindrical shell filled with a multi-phase heterogeneous liquid medium, to determine the hydrodynamic characteristics of oil and gas mixtures and the initial volume of bubbles in various water pipelines, to show that the speeds are consistent with the study of the process with complex models.

Research methods. The analytical results are based on the analytical solution of the mathematical model obtained by phenomenological methods from the general laws and principles of solid-state mechanics and physics of the two-phase liquid-shell hydrodynamic system. The study studied the hydrodynamics of the phases of acoustic (sound) waves generated by the excitation of the shell together with a heterogeneous liquid solution depending on various physical parameters.

The main points of the study. The following main scientific results were obtained:

1. A mathematical model of wave propagation as a result of monochromatic oscillations in the hydroelastic system of an infinitely long thin-walled elastic shell with two-phase liquid and gas bubbles, kinematic and dynamic contact conditions on the surfaces between the liquid medium and the shell, boundary conditions at the ends.

2. Analytical solutions of mathematical problems were obtained in the case of various physical properties of liquids - compressible, non-compressible, non-viscous, viscous Newtonian fluid, linear viscous elastic; isotropic and orthotropic thin-walled circular cross-sections of gas bubbles with a spherical fixed diameter and elastic straight lines.

3. Graphs of dynamic and kinematic characteristics of waves for different values of geometrical characteristic and experimental physical-mechanical parameters of phases and components of hydroelastic system are constructed.

Scientific novelty of the study. The main results obtained in the study are new and there are the followings:

The main results obtained in the dissertation are new and fully confirmed. The following scientific innovations were obtained in the dissertation:

Mathematical modeling of the problem of hydroelasticity of a two-phase liquid mixture with gas bubbles in an infinitely long thin shell and determination of wave characteristics were studied.

The multiphase property differs significantly from the effect of wave propagation in a shell filled with a single-phase liquid. The presented work is devoted to the study of the interaction of the liquid phase with the solid shell in a hydroaeroelastic system consisting of a cylindrical elastic shell with a bubble viscous liquid medium and to obtain new results. The solution of the problem was obtained by the method of searching as a product of functions assigned to Fourier series and numerical methods.

Theoretical and practical value of the study. The results obtained in the dissertation can be used to prepare a theoretical basis for the propagation of waves in bubbly liquids and to make practical calculations. It can be applied in the flows of complex multi-phase liquid-gas environment in deformable solids and structures, as well as in water and land transport systems, fuel and liquid transportation technological systems of flying objects. Hydrocarbons can be used in pipelines, in the analysis and design of physical and mechanical processes occurring in the vascular system of biosystems and organisms.

The issues studied in the dissertation are mainly theoretical and applied. The results obtained in the dissertation, as well as the methods used in the dissertation can be used in the oil and gas industry, in the design of the process of transportation of pipes with multi-phase heterogeneous liquids in the teaching of advanced courses.

Approbation and application. The results of the dissertation were reported at the scientific conference "Actual Problems of Mathematics and Mechanics" dedicated to the 90th anniversary of the National Leader of the Azerbaijani people Heydar Aliyev (Baku-2013), corresponding member of ANAS, Ph.D., prof. At the Republican Scientific Conference "Classical and Modern Problems of Mechanics" dedicated to the 100th anniversary of Y.A. Amanzade (Baku-2014), "Actual Problems of Mathematics and Mechanics" dedicated to the 92nd anniversary of the National Leader of the Azerbaijani people Heydar Aliyev at the scientific conference (Baku-2015), Republican Scientific Conference “Actual Issues of Theoretical and Applied Mathematics” dedicated to the 100th anniversary of academic of ANAS, Ph.D., prof. Majid Latif oglu Rasulov (Shaki-2016), International Scientific Conference “Theoretical and Applied Problems of Mathematics” (Sumgayit-2017), XVIII International Conference-Modeling and research of stability of dynamic systems ”(Kiev, Ukraine-2017), III International conference“ Development of science in information technology ”(m. Kiev | 30 version 2017),“ Actual directions of scientific and practical directions 2019 ” d.), at the IECMMA International Conference dedicated to the 100th anniversary of BSU (2019, August 26-29, Baku), "The 15th International Conference on Technical and Physical Problems of Electrical Engineering (14 - 15 October 2019 Istanbul, Turkey)," Trends and prospects for the development of science and education in the context of globalization "(May 31, 2019, issue 47).

The organization where the work was executed. Dissertation work was executed in the chair of "Theoretical mechanics and continuum mechanics" of “Mechanics-Mathematics” department of Baku State University.

Authors personal contribution. The results obtained in the dissertation belong to the applicant.

Published scientific works. The main results of the dissertation work were published in applicants 11 scientific works, 3 of which are in the journals included in the Scopus database and published in scientific publications recommended by the Higher Attestation Commission. 3 of the published articles are single-authored. In addition, the results of the dissertation were reported at 12 international and national scientific conferences, and these reports were reflected in the relevant conference materials in the form of theses.

Total volume of the dissertation work indicating separate structural units of the work in signs. The dissertation work consists of introduction, four chapters, conclusions (title page–377 signs, table of contents – 3057 signs, introduction – 34132 signs, chapter I - 60886 signs, chapter II–26775 signs, chapter III -24726 signs, chapter IV- 26076, conclusions–1840 signs) and list of references consisting of 134 names. The total volume of the dissertation work is 177869 signs.

CONTENT OF THE DISSERTATION WORK

Let us give brief review of the dissertation work consistingof an introduction, four chapters, a conclusion and a bibliography.

In the introduction we give brief summary of the dissertation, substantiates the relevance of the topic of the dissertation, reflects the main results obtained in the work and compares with the results of other works.

The first chapter consists of six subchapters. The first chapter gives the characteristics of two-phase liquids, the relationship between the liquid and the bubble, the basic assumptions in bubble liquids and a brief summary of their mathematical description. The basics of mathematical modeling of movements and processes occurring in a continuum consisting of two-phase liquid and gas bubbles in a heterogeneous environment are shown. The flow characteristics of the liquid-gas mixture in the pipes are given, in which case the cases of linear change of viscosity of the liquid with constant and pressure-dependent law are considered. The problem of transporting two-phase flows is solved and a system of complete equations is obtained. The obtained system of equations is also written in dimensionless variables.

The first subchapter of the first chapter provides an overview of the physical properties of the liquid-gas two-phase medium and the characteristics of the phase interactions. Based on the theories of Sedov, Prandtl and Levich, the determination of physical parameters and kinematic characteristics of spherical gas bubbles in different processes by interaction with liquids of different properties is analyzed, and the diameter-dependent flow velocities are determined at Newton and Stokes approximations.

The second subchapter of the first chapter analyzes the oscillation of small bubbles in the process of interaction of two-phase gas bubbles with the liquid, the phenomena of cavitation and boiling in the processes of gases with the liquid. Acoustic waves and the propagation of shock waves, depending on whether the liquids are compressed or not and the number of bubbles are reviewed. Mathematical models used in these processes, thermodynamic relationships, and possible relationships on contact surfaces are mentioned.

In the third subchapter of the first chapter the main assumptions made in mathematical modeling in multi-phase liquid media based on Nigmatulin's theory are explained for the single-velocity Rakhmatulin model. The dissertation shows the dynamic and kinematic characteristics of the liquid and gas bubble environment used in the dissertation, examples for the amount of gas bubbles.

In the fourth subchapter of the first chapter, on the basis of the equations of the theory of elasticity, different models of differential equations of motion of thin-walled elastic shells are given, taking into account the different physical properties of the materials.

The choice of orthotropic models ensures maximum adequacy to real physical processes, provided that the thin-walled pipes are schematized by means of circular cross-sections and the material has different physical properties than isotropic. Vascular systems in living organisms increase the quality of operational resources of constructive anisotropic structures with the use of composite materials.

The fifth subchapter of the first chapter analyzes the model of a two-phase liquid gas mixture in orthopedic tubes of such importance.

In the sixth subchapter of the first chapter, a mathematical model of the propagation of a wave generated by small oscillations of a liquid-gas mixture in a thin-walled pipe is given. Radial inertia is considered in the case of axesymmetric motion of the pipe with kinematic and dynamic conditions on the contact surfaces, taking into account the deformation of the pipe and the mathematical averaging of the continuity equation for the live section of the flow. In this process, the properties of a solid and a liquid multicomponent system are investigated depending on the volume concentration of the bubble.

The second chapter considers a liquid-gas system with a small number of gas bubbles as a two-phase medium. The flow of such an medium in a cylindrical tube is investigated and graphs and tables are constructed based on the numerical results obtained. Experimental and theoretical studies show that the characteristic feature of a two-phase liquid-gas medium is that the heat capacity of the carrier phase in these media is many times higher than the heat capacity of the dispersed phase.

In the first subchapter of the second chapter, a mathematical model of the viscosity of the hydrodynamic characteristics of a spherical bubble fluid shell system is selected, and the problem is simplified within the assumptions made for a multiphase fluid. These assumptions include the following relations, consisting of the equation of motion of the fluid, the averaging equation of continuity across the flow, and the rheological equation of a two-phase medium:

$$\begin{gathered} \frac{\partial u}{\partial x} + \frac{1}{\rho_0}\frac{\partial \rho}{\partial t} = -\frac{2}{R}\frac{\partial w}{\partial t} \\ \frac{\partial p}{\partial x} = -\rho_0 \frac{\partial u}{\partial t} \\ \frac{\xi}{\rho_0}\frac{\partial \rho}{\partial t} = p - a^2 \rho \end{gathered} \tag{1}$$

In these equations $u(x,t)$ - the flow rate of the mixture, $w(x,t)$ - the radial displacement of the shell, $p(x,t)$ - hydrodynamic pressure, $\rho(x,t)$ - the density of the mixture are indicated.

The square of the speed of sound propagation in a balanced environment:

$$\begin{gathered} a^2 = \frac{1}{\alpha_{20}(1-\alpha_{20})}\left(\frac{\rho_{10}}{\rho_{10}-\rho_{20}}\right)^2 \frac{p_0}{\rho_{10}} \\ \xi = \frac{4}{3}\frac{\mu(1-\alpha_{20})}{\alpha_{20}} \end{gathered} \tag{2}$$

Expression (2) - characterizes the volume viscosity, where μ the dynamic viscosity coefficient of the carrier phase, α_{20} - is the volume concentration of the bubbles, ρ_{10}, ρ_{20} - respectively, the value of the densities of the carrier and dispersed phases in equilibrium.

In the second subchapter of the second chapter, the problem is considered by throwing the inertial limit in the differential equation of the axesymmetric motion of the orthotropic shell. A differential equation with a special derivative for density is obtained. Differential equations are solved by applying the method of derivation of functions of Fourier to variables.

$$p-\frac{hE_2}{(1-\nu_1\nu_2)R^2}w=\rho_* h\frac{\partial^2 w}{\partial t^2}$$

ρ_* - density of the shell wall, E_2 - Young's Modulus, ν_1 and ν_2 - Poisson's ratio. In this expression, since the last limit is practically much smaller than the first limit, it is taken into account:

$$w=\frac{(1-\nu_1\nu_2)R^2}{hE_2}p. \tag{3}$$

Solving equations (1) and (3) in terms of density, we obtain:

$$(1+\frac{a^2}{c_0^2})\frac{\partial^2\rho}{\partial t^2}+\frac{\xi}{c_0^2\rho_0}\frac{\partial^2\rho}{\partial t^2}=a^2\frac{\partial^2\rho}{\partial x^2}-\frac{\xi}{\rho_0}\frac{\partial^3\rho}{\partial x^2\partial t}$$

The solution of the last equation is founded in the form of the product of the following functions:

$$\rho(x,t)=y(x)\exp(i\omega t)$$

The solution of the equation is reduced to the solution of a two-order, linear homogeneous differential equation:

$$y''+\delta^2 y=0$$

The solution to a homogeneous differential equation of two orders has the form:

$$y=Ae^{-i\delta x}+Be^{i\delta x}$$

By solving the problem following dependences are obtained for density, hydrodynamic pressure, velocity and displacement of the shell:

$$p(x,t)=(a^2+im_3)\left\{Ae^{-i\delta x}+Be^{i\delta x}\right\}\exp(i\omega t)$$

$$\rho(x,t)=\left\{Ae^{-i\delta x}+Be^{i\delta x}\right\}\exp(i\omega t)$$

$$u(x,t)=\frac{\delta(a^2+im_3)}{\rho_0 w}\left\{Ae^{-i\delta x}+Be^{i\delta x}\right\}\exp(i\omega t)$$

$$w(x,t)=\frac{(1-v_1v_2)R^2}{hE_1}(a^2+im_3)\left\{Ae^{-i\delta x}+Be^{i\delta x}\right\}\exp(i\omega t).$$

In the third subchapter of the second chapter is given the solution of the boundary value problem obtained in the wave process caused by the pulsasion pressure of the velocity, pressure, density and displacement of the shell in a finite-length pipe.

In common practice, pulsasion pressure is usually applied to the left end of the tube

$$p(0,t)=p^{\vee}\exp(i\omega t) \tag{4}$$

and when x = l the pressure at the other end of the pipe is zero:

$$p(l,t)=0$$

Integral constants are determined by boundary conditions and the following formulas are determined for the amplitudes of pressure, velocity, density, and displacement quantities using the Euler formulas for the semi-infinite shell:

$$|\rho|=\frac{p^{\vee}e^{-\delta_1 x}}{\sqrt{a^4+m_3^2}}$$

$$|p|=p^{\vee}e^{-\delta_1 x}$$

$$|u|=p^{\vee}\frac{e^{-\delta_1 x}}{\rho_0 w}\sqrt{\delta_0^2+\delta_1^2} \tag{5}$$

$$|w|=p^{\vee}e^{-\delta_1 x}\frac{(1-v_1v_2)R^2}{hE_2}$$

The issue considered in the fourth subchapter of the second chapter shows the determination of the dynamic parameters of the fluid and the characteristics of the movement through the flow of the fluid.

$$|\rho| = \frac{Q^v \cdot \rho_0 \cdot \omega}{\pi R^2 \sqrt{(\delta_1^2 + \delta_0^2)\cdot(a^4 + m_3^2)}} \cdot \exp(-\delta_1 x)$$

$$|p| = \frac{Q^v \cdot \rho_0 \cdot \omega}{\pi R^2 \sqrt{(\delta_1^2 + \delta_0^2)}} \cdot \exp(-\delta_1 x) \qquad (6)$$

$$|w| = \frac{Q^v \cdot \omega}{2\pi R c_0^2 \sqrt{(\delta_1^2 + \delta_0^2)}} \cdot \exp(-\delta_1 x)$$

$$|u| = \frac{Q^v \cdot \omega}{\pi R^2} \cdot \exp(-\delta_1 x)$$

Graphs were made for four liquids containing air bubbles to show that the amplitudes of the characteristics found by the flow varied depending on the unit volume of the bubbles.

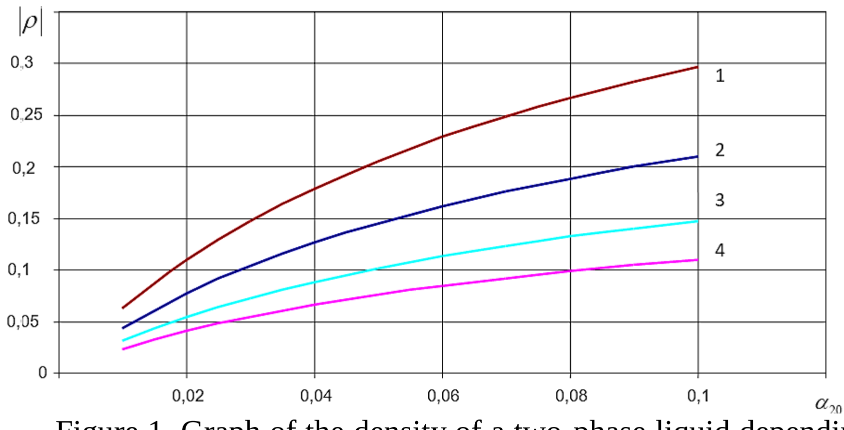

Figure 1. Graph of the density of a two-phase liquid depending on the number of bubbles; 1- glycerin, 2-water, 3-ethanol, 4-light oil.

The third chapter is devoted to the pulsating motion of a non-compressible bubble fluid in a viscous-elastic shell. Mathematical modeling of the propagation of waves generated in the hydroelastic system is carried out taking into account the viscous elastic friction of the environment. For comparison, given the basic assumptions of the one-dimensional theory for a straight-axis elastic isotropic shell of small oscillations of an ideal fluid inside, the problem is the pulsar motion of an incompressible fluid inside a semi-cylindrical orthotropic coating and the viscous motion of a viscous bubble fluid in a viscous-elastic material.

In the first subchapter of the third chapter, a one-dimensional motion model is considered, and the equations of mass storage and momentum in the process of movement of a semicircular tube and a two-phase solution inside are written. The model is constructed taking into account the hereditary elastic model of environmental impact.

In the expression of the equation of motion of the pipe wall, taking into account the dimensional parameter of the core and elastic stiffness, which expresses the viscous friction within the theory of hereditary elasticity:

$$p = \rho_* h \frac{\partial^2 w}{\partial t^2} + G\left\{\frac{\partial w}{\partial t} - \int_{-\infty}^{t} G_0(t-\tau)\frac{\partial w(x,\tau)}{\partial \tau}d\tau\right\} + \frac{Eh}{R^2(1-\nu^2)}w$$

The basic hydrodynamic parameters of the environment are determined by applying the method of separation to variables by solving the equation that characterizes the fluid, the equation of the shell. Graphs of the dependence of the wave velocity on the stiffness coefficient and the extinction wave velocity are plotted.

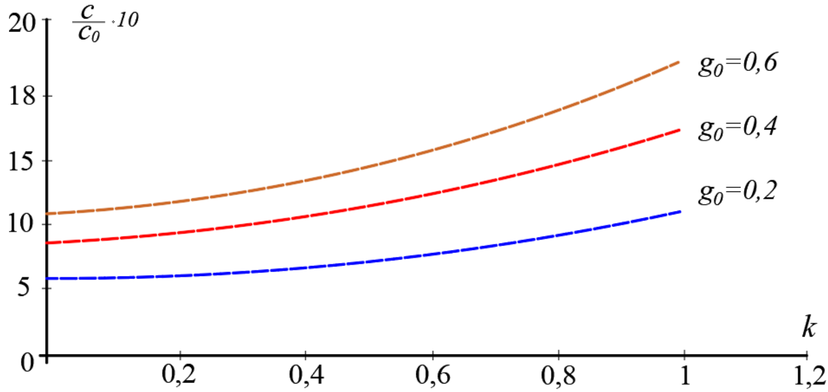

Figure 2. Dependence of wave velocity on stiffness coefficient

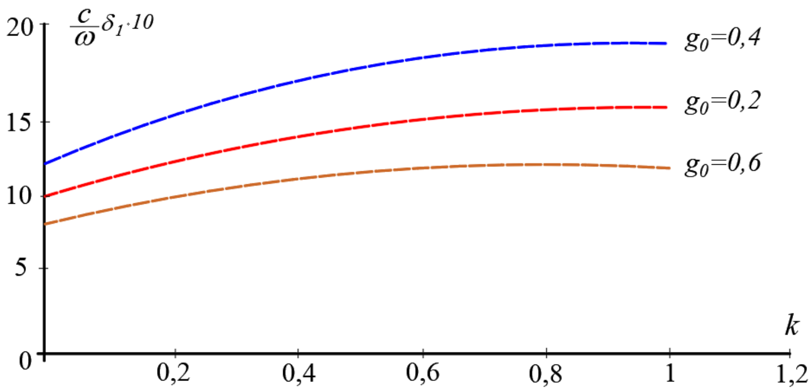

Figure 3. Dependence of extinction on wave speed

In the second subchapter of the third chapter, the analysis of waves generated in small excitations is carried out, taking into account the elastic friction in the viscous elastic tube and the viscous elastic friction in the semiconducting orthotropic tube. Depending on the radial displacement of the shell, a special derivative differential equation is obtained. The solution of the equation is sought by the method of functions assigned to Fourier variables. Analytical formulas for pressure are obtained and characteristic curves are constructed for specific physical, experimental and design parameters.

In the third subchapter of the third chapter, the pulsasion motion of a non-compressible bubble fluid in a viscous-elastic shell is considered. The linear inheritance operator is chosen as the model. The solution of the problem is sought in the class of functions assigned to the variables.

The equation of motion of a pipe takes the following form, taking into account the expression of the hereditary operator:

$$p = \frac{hE}{(1-\nu^2)R^2}\left\{ w - \int_{-\infty}^{t} \Gamma(t-\tau) w \, d\tau \right\} + \rho_* h \frac{\partial^2 w}{\partial t^2}$$

The basic parameters are determined as a result of the joint solution of the equations of motion of the viscous-elastic shellwith the equation of motion of the viscous-elastic shell:

$$\rho(x,t) = \beta \exp\left[\left(i(\omega t - \delta x)\right)\right]$$

$$p(x,t) = \beta(a^2 + im_3)\exp\left[\left(i(\omega t - \delta x)\right)\right]$$

$$u(x,t) = -\beta \frac{\delta}{\rho_0 \omega}(a^2 + im_3)\exp\left[\left(i(\omega t - \delta x)\right)\right] \qquad (7)$$

$$w(x,t) = \beta \frac{(1-v^2)R^2}{(1-\alpha)hE}(a^2 + im_3)\exp\left[\left(i(\omega t - \delta x)\right)\right]$$

Here, $\alpha = \int_0^\infty \Gamma(\theta) e^{-i\omega\theta} d\theta,\ \beta = \frac{P^v}{a^2 + im_3}$

At the end of the subchapter, the dependence of the wave decrement on the relaxation nucleus of the cover and the dependence of the wave speed on the relaxation core of the cover in a two-phase environment are plotted.

The fourth chapter of the dissertation is devoted to the mathematical modeling of the propagation of waves generated by the excitation of a liquid at low velocities from the monochromatic center in a cylindrical shell in a hydroelastic system consisting of a viscous Newtonian fluid and gas bubbles at rest.

In the first subchapter of the fourth chapter, the mathematical modeling of the problem of hydrodynamics in the process of wave propagation of the hydroelastic system of a liquid-gas two-phase medium in an elastic isotropic cylindrical shell of infinite length is described in full. First, mathematical modeling of the propagation of waves in a liquid-gas mixture is carried out and the problem is solved. The Navier-Stokes linear equation, the equation of continuity in a linear problem, and the equation of state of a liquid medium are given in the following form:

$$\rho_f \frac{\partial \vec{u}}{\partial t} = -gradp + \frac{1}{3}\mu\, graddiv\vec{u} + \mu\nabla^2\vec{u} \tag{8}$$

$$\frac{\partial \rho}{\partial t} + \rho_f\, div\vec{u} = 0 \tag{9}$$

$$\frac{\partial p}{\partial \rho} = a_f{}^2 \tag{10}$$

The line of dependence between the hydrodynamic pressure and deformation rate components of the stresses in motion in a two-phase liquid-gas environment is for a viscous Newtonian fluid:

$$\sigma_{ik} = \left(-p + \lambda_1 div\vec{u}\right)\delta_{ik} + 2\mu_1 e_{ik} \tag{11}$$

$$e_{ik} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i}\right) \tag{12}$$

Here, the wave equation for pressure is obtained in the form of the following differential equation

$$\frac{1}{a_f^2}\frac{\partial^2 p}{\partial t^2} = \nabla^2\left(p + \frac{4\mu}{3\rho_f a_f^2}\frac{\partial p}{\partial t}\right). \tag{13}$$

The differential equations of motion of the shell for the axisymmetric case are given by the Kirchhoff-Lyav hypothesis as follows

$$\rho_* h\frac{\partial^2 w_r}{\partial t^2} = p_t\left(1 - \frac{h}{2R}\right) + \frac{Eh}{2(1+\nu)}\frac{\partial^2 w_r}{\partial x^2} - \frac{Eh}{(1-\nu^2)}\left\{\frac{w_r}{R^2} + \frac{\nu}{R}\frac{\partial w_r}{\partial x}\right\} \tag{14}$$

$$\rho_* h\frac{\partial^2 w_x}{\partial t^2} = q\left(1 - \frac{h}{2R}\right) + \frac{Eh}{(1-\nu^2)}\left\{\frac{\partial^2 w_x}{\partial x^2} + \frac{\nu}{R}\frac{\partial w_r}{\partial x}\right\} \tag{15}$$

The closed mathematical system of hydroelastics or the mathematical model of the problem is the system of equations (8), (13), (14) and (15). These are the unknown functions of the system of equations p, u_r, u_x, w_r, w_x. Appropriate boundary and contact conditions must be provided for the identification of these unknowns. To solve a system of equations, the appropriate dynamic contact conditions and kinematic boundary conditions must be met:

$$p_t = -\sigma_{rr}\big|_{r=R-h/2} = -\left(-p - \frac{2}{3}\mu\, div\,\vec{u} + 2\mu\frac{\partial u_r}{\partial r}\right)$$

$$q = -\sigma_{rx}\big|_{r=R-h/2} = -\mu\left(\frac{\partial u_x}{\partial r} + \frac{\partial u_r}{\partial x}\right)$$

Also, the kinematic boundary conditions along that surface are true:

$$u_r\big|_{r=R} = \frac{\partial w_r}{\partial t}\bigg|_{r=R},$$

$$u_x\big|_{r=R} = \frac{\partial w_x}{\partial t}\bigg|_{r=R}.$$

In the second subchapter of the fourth chapter, we look for the solution of the linear wave equation of a liquid-gas homogeneous viscous medium by pressure as a product of the functions assigned to the variables as follows:

$$p = \mathrm{Re}\left\{p^*(r)e^{kx+i\omega t}\right\}$$

According to the physical nature of the problem, since $p^*(r)$- has a finite value when $r = 0$, it must be $C_0=0$, so

$$p^*(r) = p_0 J_0(\lambda r)$$

If we look for a solution from (8) and (9) in the same way for the velocities of the particles of a liquid gas mixture,

$$u_r = \mathrm{Re}\left\{u_r^*(r)e^{kx+i\omega t}\right\} \tag{16}$$

$$u_x = \mathrm{Re}\left\{u_x^*\, e^{kx+i\omega t}\right\} \tag{17}$$

the following solutions are obtained:

$$u_r^* = AJ_1(\beta r) - \frac{p_0\lambda\left(1 + i\dfrac{\omega\mu}{3\rho_f a_f^2}\right)}{\mu\left(\beta^2 - \lambda^2\right)} J_1(\lambda r) \tag{18}$$

$$u_x^* = BJ_0(\beta r) - \frac{ip_0 k\left(1 + i\dfrac{\omega\mu}{3\rho_f a_f^2}\right)}{\mu\left(\beta^2 - \lambda^2\right)} J_0(\lambda r)$$

here

$$\beta^2 = -\left(k^2 + i\frac{\rho_f \omega}{\mu}\right)$$

Is the complex quantity.

Considering the state equation of a liquid medium (16) and (17) in the equation of continuity:

$$B = -\frac{i\beta}{k}A \tag{19}$$

We express the obtained solutions in dimensionless quantities. *A*, *B* and P_0 are constants and cannot be determined from the solution of the problem.

$$\bar{p}^* = \bar{p}_0\, J_0(\bar{\lambda}\bar{r})$$

$$\bar{u}_r^{\,*} = J_1(\bar{\beta}\bar{r})\bar{A} - i\frac{\bar{\lambda}\bar{\omega}J_1(\bar{\lambda}\bar{r})}{K^2 + \bar{\lambda}^2}\bar{p}_0$$

$$\bar{u}_x^{\,*} = i\frac{\bar{\beta}}{K}J_0(\bar{\beta}\bar{r})\bar{A} - \frac{K\bar{\omega}J_0(\bar{\lambda}\bar{r})}{K^2 + \bar{\lambda}^2}\bar{p}_0 \tag{20}$$

$$\bar{\beta}^2 = -(K^2 + i\frac{\bar{\omega}}{\bar{\mu}})$$

$$\bar{\lambda} = -K^2 + \frac{\bar{\omega}^2}{\left(1 + i\frac{4\bar{\mu}\bar{\omega}}{3}\right) - K^2}$$

Here a_t - is the speed of sound propagation in the pipe.

The equation of the cover is dimensionless:

$$\frac{\partial^2 \bar{w}_r}{\partial \bar{t}^2} = \bar{p}_t\left(1 - \frac{\bar{h}}{2}\right) + \frac{1}{\eta}\frac{\partial^2 \bar{w}_r}{\partial \bar{x}^2} - \bar{w}_r - v\frac{\partial \bar{w}_x}{\partial \bar{x}}$$

$$\frac{\partial^2 \bar{w}_x}{\partial \bar{t}^2} = \bar{q}\left(1 - \frac{\bar{h}}{2}\right) + \frac{\partial^2 \bar{w}_x}{\partial \bar{x}^2} + v\frac{\partial \bar{w}_r}{\partial \bar{x}}\,. \tag{21}$$

The displacement of the shell points is sought by the mode of excited movement of the fluid inside the shell. Therefore, the radial and axial components of displacements are sought as a product of the radial coordinate function and the axial coordinate and time together.

$$\overline{w}_r = \overline{w}_r{}^* e^{i\left(Kx+\overline{\omega}_t \bar{t}_t\right)}$$
$$\overline{w}_x = \overline{w}_x{}^* e^{i\left(Kx+\overline{\omega}_t \bar{t}_t\right)} \qquad (22)$$

The following expressions are obtained for the displacement components of the differential equations of the shell, taking into account the contact conditions for normal and frictional forces on the inner surface of the coating.

$$\overline{w}_r{}^* = \overline{p}_t{}^* \frac{1-\overline{h}/2}{\Phi} + \overline{q}^* \frac{\theta}{\Phi}$$
$$\overline{w}_x{}^* = \overline{p}_t{}^* \frac{\left(-iK(1-\overline{h}/2)\nu\right)}{\Phi\left(-K^2+\overline{\omega}_t{}^2\right)} + \overline{q}^* \frac{\left\lfloor -i\nu K\theta + \Phi(1-\overline{h}/2)\right\rfloor}{\Phi\left(-K^2+\overline{\omega}_t{}^2\right)}$$

here

$$\Phi = 1-\overline{\omega}_t{}^2 + \frac{\nu^2 K^2}{\left(-K^2+\overline{\omega}_t{}^2\right)} - \frac{\xi K^2\left(-K^2+\overline{\omega}_t{}^2\right)}{1-\eta\xi\left(-K^2+\overline{\omega}_t{}^2\right)}$$
$$\theta = 1-\overline{\omega}_t{}^2 + \frac{i\nu K\left(1-\overline{h}/2\right)}{\left(-K^2+\overline{\omega}_t{}^2\right)} - \frac{i\overline{h}K\left(1-\overline{h}/2\right)}{2\left(1-\eta\xi\left(-K^2+\overline{\omega}_t{}^2\right)\right)}$$

For linear homogeneous algebraic equations with unknown $\overline{A}$ and $\overline{P}$ coefficients of the kinematic boundary condition are obtained.

$$c_{11}\overline{A} + c_{12}\overline{p}_0 = 0$$
$$c_{21}\overline{A} + c_{22}\overline{p}_0 = 0$$

For a system of equations to be a non-trivial solution, the determinant must be zero.

In the third subchapter of the fourth chapter, the mathematical modeling of wave propagation in a hydroeloastic system and the dispersion equation solved from the condition that determinant equal to zero in the analytical formulas obtained for fluid pressure, fluid particle velocities and shell displacements, taking into account boundary-contact conditions. The solution process is brought to the solution of the algebraic equation depending on the ω frequency of excitation of the wave number *K*.

As the solutions of the dispersion equations of the second type of waves or ω_R are the real positive numbers $\omega = \omega_R$ it will be like this $\omega > 0$, $\omega_i = 0$. We get complex numbers $K(\omega)$ for the arbitrary ω. These waves are created by the external source when $x = 0$ in the fluid-shell system. On the waves excitations in the balanced liquid that is inside the shell, $K_i(\omega) > 0$ characterizes the decrement of oscillations or the growth of the oscillations towards the axis. In the obtained solutions if it is like this $K_R(\omega) < 0$, the phase of the oscillations is spreading at $a(\omega)$ speed $a(\omega) > 0$.

There are damped acoustic excitations that extend along the wave length on the oscillations in the forms of one-speed motion of the two-phase medium bubble fluids.

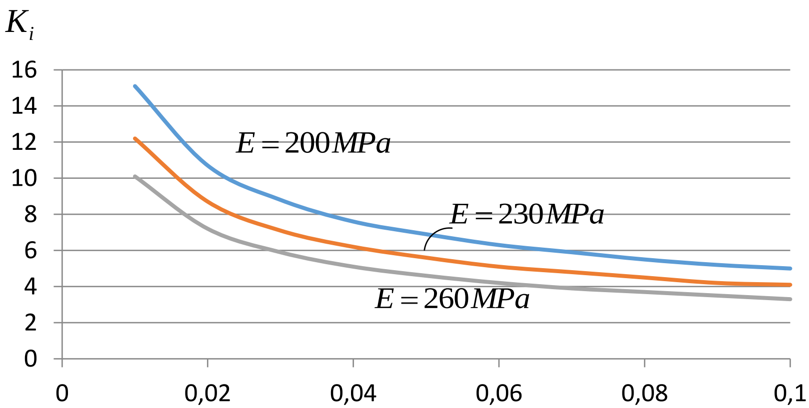

Fig.4. For the points of $E = 200 MPa$; $E = 230 MPa$; $E = 260 MPa$, $\rho_* = 920 kq/m^3$ (for high pressure polyethylene) the dependence of the K_i on the concentration of the bubbles;

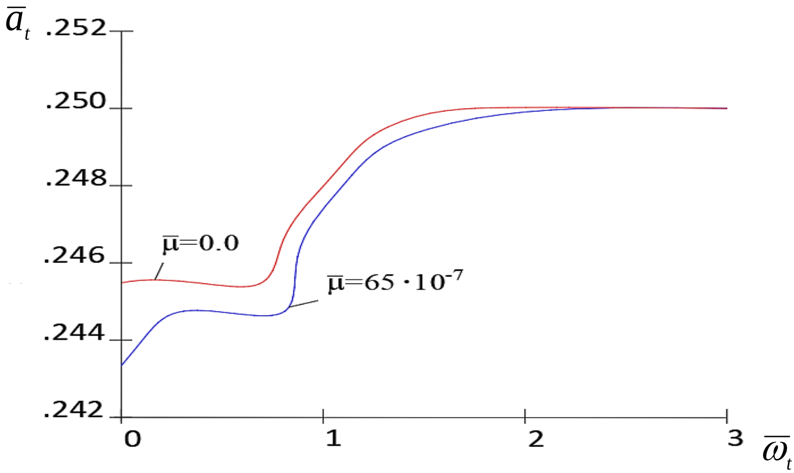

Fig.5. The dependence of the velocity ot the phases of the mixture on the frequency

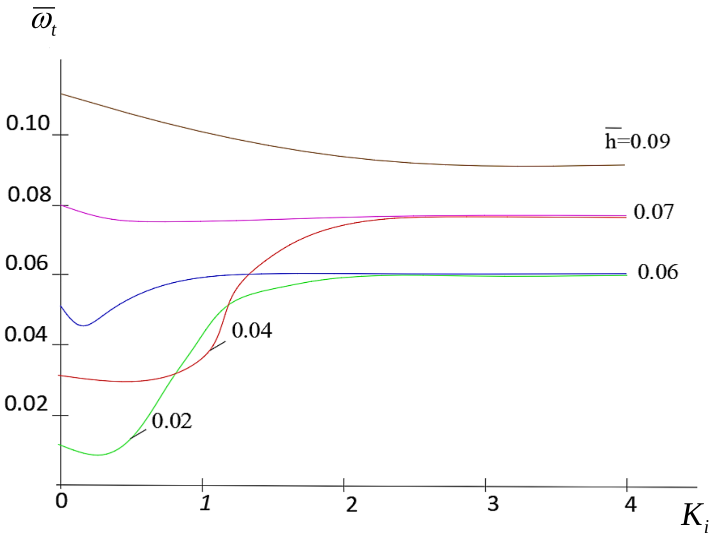

Fig.6. Dependence of the frequency values on the shell thickness

Fig.6. shows the effect of the influence of the values of the wall thickness of the shell on the value of the dimensionless vibration frequency of the shell filled with a gas-liquid mixture with large

bubbles. The result reflects the fact that the stiffness of the shell increases with increasing thickness, and therefore the frequency of the thickened shell is higher in magnitude than the frequency of the shell of moderate thickness. This result is typical for objects of biomechanics research - for growing living organisms and natural plants.

Conclusion

1. In the flow of a two-phase viscous fluid in deformable elastic shells, it is determined that the uniform volume of gas bubbles in the medium increases depending on the δ_0- concentration coefficient. . Numerical calculations reflect the variations in the range of $\alpha_2 = 0.01 \div 0.1$ the shell-two-phase environment system.
2. In the process of movement of a two-phase liquid medium (non-compressible linear viscous liquid and compressed gas bubble) in an elastic cylindrical tube, the speed of propagation of waves in the tube decreases, and the effect of fluid viscosity on the movement characteristics is small.
3. It has been shown that the velocity of the waves generated in a pulsar flow in a cylindrical viscous elastic tube of a viscous incompressible liquid and a bubble decreases with the volume concentration of the bubble and the relative length of the tube, and the flow rate of the mixture increases.
4. A comparison of the results of the calculation shows that the dispersion of the waves transmitted through the system of “orthotropic elastic tube-viscous liquid mixture” depends on the physical properties of the walls of flexible elastic shells- the viscosity of the liquid. At high frequencies, the elasticity of the shell is predominant and at low frequencies, wave propagation is prominent.
5. Depending on the physical parameters of the environment, the values of velocity, displacement, density and hydrodynamic pressure change significantly as the volume coefficient of bubbles in a two-phase medium increases.
6. It has been found that an increase in the density of the carrier phase and a flow rate from a stationary value, starting from that stationary value, a decrease in the values of the pressure drop and the radial displacement of the shell wall.
7. It has been shown that in a large-length cylindrical elastic cover filled with a liquid at rest, the speed of propagation of a wave in the positive direction of the axis during the propagation of small

excitations is converted into phase velocity. This corresponds to the mode in which the excitation amplitude is in the direction of phase velocity and the oscillation phase in the direction of propagation.

The main results of the presented thesis has been published in following works:

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